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# The lightest Higgs mass in supersymmetric models with extra dimensions <sup>\*</sup>

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## Abstract

In the four-dimensional supersymmetric standard model extended with gauge singlets the lightest Higgs boson mass,  $M_H$ , has an important contribution proportional to the squared of the superpotential coupling  $\lambda$  of singlets to Higgs fields,  $\lambda S H_1 \cdot H_2$ . The requirement of perturbativity up to the unification scale yields an upper bound on  $M_H \sim 140$  GeV. In extensions to theories with (longitudinal) extra dimensions at the TeV where such coupling exists and massive Kaluza-Klein states fall into  $N = 2$  representations, if either of the Higgs or singlet fields live in the bulk of the extra dimensions, the  $\beta$ -function of  $\lambda$  is suppressed due to the absence of anomalous dimension of hypermultiplets to leading order. This implies a slower running of  $\lambda$  and an enhancement of its low energy value. The  $M_H$  upper bound increases to values  $M_H \lesssim 165$  GeV.

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LEP unification of the gauge couplings in the minimal supersymmetric extension of the Standard Model (MSSM), possibly including gauge singlets (NMSSM), is the only “hint” of new physics at present accelerators. In fact, running the gauge couplings from their LEP values ( $M_Z$ ) to high energies with the (N)MSSM four-dimensional (4D)  $\beta$ -functions one finds a remarkable unification of the  $SU(3) \times SU(2) \times U(1)$  couplings at a scale  $M_U^{4D} \simeq 2 \times 10^{16}$  GeV. Furthermore, the existence of a light Higgs with a tree-level upper bound (independent of the values of the soft masses of all other scalar partners) on its mass,  $M_H$ , is one of the most solid, and model independent, predictions of supersymmetric theories. In particular for the MSSM there exists the tree-level bound  $M_H \leq M_Z |\cos 2\beta|$  which, after inclusion of radiative corrections yields, for the experimental value of the top-quark mass, an absolute upper bound  $M_H \lesssim 125$  GeV [1, 2]. For the NMSSM, assuming a superpotential coupling of gauge singlets  $\vec{S}$  as

$$W = \vec{\lambda} \vec{S} H_1 \cdot H_2 , \quad (1)$$

one finds the tree level upper bound for the lightest Higgs mass:

$$M_H^2 \leq \left( \cos^2 2\beta + \frac{2\vec{\lambda}^2 \cos^2 \theta_W}{g_2^2} \sin^2 2\beta \right) M_Z^2 . \quad (2)$$

Imposing perturbativity up to  $M_U^{4D}$ , and including radiative corrections, an upper bound for the lightest Higgs mass as  $M_H \lesssim 140$  GeV [3] is found.

Being the above bounds for the Higgs mass rather restrictive, the corresponding models could be ruled out at the next run of proton colliders (TeVatron, LHC) if the Higgs is not seen in the corresponding ranges, irrespectively of any direct search for supersymmetric partners. A way of increasing the absolute upper bound on the lightest Higgs mass in the NMSSM, without spoiling the unification properties of the gauge couplings, was proposed by introducing extra lepton and quark fields in  $SU(5)$  representations, plus complex conjugates to avoid anomalies appearance. These models neither spoil unification nor change the unification scale, but they increase the value of the unification coupling,  $\alpha_U^{4D}$ , moving it towards the non-perturbative regime. Increasing the gauge couplings all the way between  $M_Z$  and  $M_U^{4D}$  slows down the rate of running of  $\vec{\lambda}$ , which can then acquire larger initial values while remaining perturbative up to  $M_U^{4D}$ . Obviously this effect increases the upper bound on the lightest Higgs mass. By adding extra matter at scales  $\gtrsim 1$  TeV in the representation  $\mathbf{5} + \bar{\mathbf{5}}$  it was shown that  $M_H \lesssim 155$  GeV [4, 5], that can be considered as the absolute upper bound.

In this Letter we will propose another possible way of increasing the absolute bound on  $M_H$ , without neither spoiling the unification properties of the MSSM nor moving  $\alpha_U$  to the non-perturbative regime. It is based on the existence of extra dimensions at the TeV scale, where the gauge bosons and some of the Higgs and/or matter fields live. By using

this genuine extra-dimensional mechanism we will see that the upper bound on  $\vec{\lambda}$  (i.e. on  $M_H$ ) can be considerably enhanced with respect to the NMSSM case. In the simplest example that we will explicitly work out, based on one-extra dimension compactified on the orbifold  $S^1/\mathbb{Z}_2$ , the absolute upper bound is increased to  $M_H \lesssim 165$  GeV. Finally we will prove how robust this result is versus the number of extra dimensions.

The existence of large (TeV) extra dimensions feeling gauge interactions has been shown in a large class of string theories, including type I and type IIB vacua. The possibility of lowering the compactification scale  $M_c \equiv 1/R$  to the TeV was first proposed in Ref. [6], while the option of lowering the string (quantum gravity) scale,  $M_s$ , was suggested in Ref. [7]. Having the fundamental scale close to the TeV implies that some ( $\delta$ ) extra dimensions “parallel” to the 4D space can appear at the TeV while other, “transverse”, ones (where gravity lives) can be much larger (sub-millimeter dimensions) [8].

If  $M_s \gg M_c$  there are threshold corrections for the couplings at  $M_c$ , arising from loops of Kaluza-Klein (KK) excitations, that can be interpreted as a power-law running [9] and can lead to unification of gauge couplings at some scale  $M_U$  that we will identify with the string scale [10]<sup>1</sup>. Depending on the fields living in the bulk of the extra dimensions (apart from the gauge fields that we always assume to live in it), we can have unification of gauge couplings, but at a much lower scale than  $M_U^{4D}$ .

In this work we will first focus on the case of just one “parallel” dimension ( $\delta = 1$ ) at the TeV scale, where gauge bosons and some of the Higgs and matter multiplets live. As we will see the results are easily generalized to the case  $\delta > 1$ . We consider a 5D theory compactified on  $S^1/\mathbb{Z}_2$  [11, 12]. Vector fields in the bulk are in  $N = 2$  vector multiplets,  $\mathbb{V} = (A_\mu, \lambda_1; \Phi, \lambda_2)$ <sup>2</sup>, while matter fields in the bulk are arranged in  $N = 2$  hypermultiplets,  $\mathbb{H} = (z_1, \psi_R; z_2, \psi_L)$ .  $\mathbb{Z}_2$  is acting as a parity in the fifth dimension,  $x_5 \rightarrow -x_5$  and, with an appropriate lifting to spinors and  $SU(2)_R$  indices, we can decompose  $\mathbb{V}$  and  $\mathbb{H}$  into *even*,  $(A_\mu, \lambda_1)$  and  $(z_1, \psi_R)$ , and *odd*,  $(\Phi, \lambda_2)$  and  $(z_2, \psi_L)$ ,  $N = 1$  superfields. After the  $\mathbb{Z}_2$  projection the only surviving zero modes are in  $N = 1$  vector and chiral multiplets. If the chiral multiplet  $(z_1, \psi_R)$  is not in a real representation of the gauge group, a multiplet of opposite chirality localized in the 4D boundary  $(\tilde{z}, \tilde{\psi}_L)$  can be introduced to cancel anomalies. In order to do that  $\mathbb{Z}_2$  must have a further action on the boundary under which all boundary states are odd<sup>3</sup>. This action can be defined as  $(-1)^{\varepsilon_i}$  for the chiral multiplet  $X_i$ , such that  $\varepsilon_i = 1$  (0) for  $X_i$  living in the boundary (bulk). The MSSM is then made up of zero modes of fields living in the 5D bulk and

<sup>1</sup>We also expect in general (model dependent) string threshold corrections. However in the regime  $M_s \gg M_c$  they are subleading compared to the contribution from the massive KK modes and will not be considered here.

<sup>2</sup>Where  $i = 1, 2$  transform as  $SU(2)_R$  indices and the complex scalar  $\Phi$  is defined as,  $\Phi \equiv A_5 + i\Sigma$ .

<sup>3</sup>We thank C. Kounnas for interesting discussions about this point.

chiral  $N = 1$  multiplets in the 4D boundary, at localized points of the bulk. The Higgs and matter sector ( $H_1, H_2, Q, U, D, L, E$ ) will then be allowed to live, either in the bulk as independent hypermultiplets or at the boundary as chiral multiplets. In either case, after the  $\mathbb{Z}_2$  projection, they lead to the supermultiplet structure of the MSSM.

The general MSSM unification condition in the presence of extra dimensions was obtained in Ref. [13] as:

$$2N_E + 5N_U - 7N_Q + 3(N_D - N_L) - 3(N_{H_1} + N_{H_2}) + 2 - 32T_0 - 20T_1 = 0 , \quad (3)$$

where  $N_X$  ( $X = H_1, H_2, Q, U, D, L, E$ ) is the number of  $X$  hypermultiplets. We have considered the possibility of  $T_Y$  pairs of hypermultiplets in the bulk which transform as  $(\mathbf{1}, \mathbf{3})_{|Y|}$  under  $SU(3) \times SU(2) \times U(1)$ . The hypermultiplets which are not included in the usual generations, or transform as real representations of the gauge group, must be considered in pairs to cancel the anomalies of the zero modes after the  $\mathbb{Z}_2$  projection. We assume in Eq. (3) all gauge bosons living in the bulk, which is the natural possibility to achieve unification of gauge couplings [14]. The relationship between the unification scales  $M_U$  and  $M_U^{4D}$  is given by,

$$\frac{M_U}{M_c} = \rho \log \frac{M_U^{4D}}{M_c} , \quad (4)$$

where

$$\rho = 14/[10 + 3N_E - N_L - (N_{H_1} + N_{H_2}) - 7N_Q + 4N_U + N_D - 10T_0 - 2T_1] .$$

The meaning of Eq. (4) is clear: in models with power-law running (linear in the case of a single extra dimension) the evolution of gauge couplings is much faster than in models with logarithmic running. The former ones run less (left-hand side of (4)) than the latter ones (right-hand side of (4)) and, as a consequence, unify earlier. In models with  $\rho = 1$  the different evolution rates are exactly compensated by different unification scales. Therefore MSSM-like unification is more natural in models with  $\rho = 1$ , as we will require in this work <sup>4</sup>.

In what follows we will be concerned with higher dimensional models that unify as the MSSM, Eq. (3), and where there are superpotential couplings on the 4D boundary, as in Eq. (1), that can increase the lightest Higgs tree-level mass as in Eq. (2). In general, for the existence of a superpotential coupling as  $f^{i_1 \dots i_N} X_{i_1} \cdots X_{i_N}$ , the orbifold selection rule

$$(-1)^{\varepsilon_{i_1} + \dots + \varepsilon_{i_N}} = 1 \quad (5)$$

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<sup>4</sup>Of course one can find unifying models satisfying (3) with  $\rho < 1$ . They will not be considered in detail in this work since they require more extra matter and easily conflict the original condition that  $M_U \gg M_c$ . In fact the constraint  $\rho = 1$  is more accurate in models with  $\delta > 1$ . We will comment later on about these possibilities.

is required [15, 16]. This condition implies, for renormalizable couplings, that  $\varepsilon_i + \varepsilon_j + \varepsilon_k = 2$ <sup>5</sup>. Electroweak precision measurements [17, 16], as well as direct searches [18], constrain, due to the presence of KK excitations of Standard Model gauge bosons, the value of the compactification scale  $M_c$  in each case, providing bounds which depend on the chosen values of the parameters  $\varepsilon_i$  and  $\tan\beta$  [16]. Those bounds will be taken into account in the subsequent analysis.

In the considered models the gauge bosons are living in the bulk. They can be used to break supersymmetry by their interactions with the hidden wall [19], or by the compactification mechanism [12]. In either case they induce a gauge mediated supersymmetry breaking (GMSB) scenario. Our subsequent analysis of the lightest Higgs mass is not sensitive to the details of supersymmetry breaking, or spectrum, that will consequently be of no concern in this paper. On the other hand the Higgs fields  $H_1$  and  $H_2$  can, either of them, live in the bulk of the extra dimension or in the 4D boundary. In each case the consequences for the Higgs mass will be different, as we will see.

If both  $H_1$  and  $H_2$  are living in the bulk ( $N_{H_1} = N_{H_2} = 1$ ) and superpotential Yukawa couplings are assumed for leptons and quarks, Eq. (5) can be satisfied provided that all three generation matter multiplets live in the 4D boundary. Unification as in the MSSM, with  $\rho = 1$ , can be easily performed by introducing two extra  $E$ -like hypermultiplets at the compactification scale,  $\Delta N_E = 2$ <sup>6</sup>. This model was introduced in Ref. [20] and it is contained in condition (3). In this case no coupling as in Eq. (1) is consistent with Eq. (5), neither for singlets in the bulk or in the 4D boundary<sup>7</sup>. The second term in Eq. (2) is zero and the lightest Higgs mass bound coincides with that of the MSSM, i.e. the first term in Eq. (2).

If the  $H_1$  and  $H_2$  Higgs fields live in the 4D boundary ( $N_{H_1} = N_{H_2} = 0$ ) and Yukawa couplings for leptons and quarks are assumed, it follows from Eq. (5) that some matter multiplets must live in the 4D boundary while others must propagate in the 5D bulk. A discussion of the different possibilities was performed in Ref. [16], where lower bounds for the different cases were found from electroweak precision measurements. There exists a minimal model with MSSM-like unification and  $\rho = 1$ : right-handed quarks and leptons live in the bulk, left-handed quarks and leptons in the 4D boundary,  $N_U = N_D = N_E = 3$ , and there is also a pair of zero-hypercharge triplets at the scale  $M_c$  which live in the bulk,  $T_0 = 1$ . The presence of triplets at  $M_c$  plays no role for the lightest Higgs mass bound,

<sup>5</sup>The possibility  $\varepsilon_i + \varepsilon_j + \varepsilon_k = 0$  is not excluded by the orbifold group action but it is suppressed, with respect to the former one, by an extra factor  $\sim 1/(M_s R) \ll 1$  [16].

<sup>6</sup>A supersymmetric mass term for pairs of hypermultiplets can be easily obtained by a supersymmetry preserving compactification [12].

<sup>7</sup>As previously indicated the coupling (1) is consistent with the orbifold action (5) for singlets in the bulk, but this coupling is expected to be highly suppressed and its contribution in (2) expected to be negligible.

since they are introduced merely for the purpose of unification. Had we not introduced them the model would not unify but the corresponding Higgs mass bounds would be only slightly modified. In this case  $\vec{S}$ -singlet(s) in the bulk can couple to  $H_1 \cdot H_2$  with the coupling  $\vec{\lambda}$  in the superpotential (1) and the tree-level Higgs mass bound (2) applies.

As in 4D models, in order to evaluate the absolute upper bound on the lightest Higgs mass we need to compute the upper bound on  $\vec{\lambda}$  at low energy by imposing perturbativity of the theory up to the unification scale  $M_U$ . The relevant Yukawa couplings are in this case  $h_t$ ,  $h_b$  (the top and bottom quark Yukawa couplings) and  $\lambda$  (we will assume one singlet or orientate  $\vec{\lambda}$  along one particular direction). For  $M_Z \leq \mu \leq M_{\text{SUSY}}$ <sup>8</sup> the beta functions for gauge and Yukawa couplings are those of the Standard Model, and for  $M_{\text{SUSY}} \leq \mu \leq M_c$  those of the MSSM. For  $\mu \geq M_c$  the extra dimension (KK-modes) is felt by the couplings and the one-loop  $\beta$ -functions for Yukawa couplings can be written as:

$$\begin{aligned} 16\pi^2 \beta_\lambda &= (\gamma_S^S + \gamma_{H_1}^{H_1} + \gamma_{H_2}^{H_2}) \lambda \\ 16\pi^2 \beta_{h_t} &= (\gamma_Q^Q + \gamma_U^U + \gamma_{H_2}^{H_2}) h_t \\ 16\pi^2 \beta_{h_b} &= (\gamma_Q^Q + \gamma_D^D + \gamma_{H_1}^{H_1}) h_b . \end{aligned} \quad (6)$$

The one-loop wave-function renormalization is given by <sup>9</sup>

$$\gamma_X^X \equiv \{e^t \varepsilon_X + (1 - \varepsilon_X)\} \tilde{\gamma}_X^X , \quad (7)$$

where  $t$  is related to the RGE scale  $\mu$  by  $t \equiv \log(\mu/\mu_0)$ , and [21, 22]

$$\begin{aligned} \tilde{\gamma}_S^S &= 2\lambda^2 \\ \tilde{\gamma}_{H_1}^{H_1} &= -\frac{3}{2}g_2^2 - \frac{3}{10}g_1^2 + 3h_b^2 + \lambda^2 \\ \tilde{\gamma}_{H_2}^{H_2} &= -\frac{3}{2}g_2^2 - \frac{3}{10}g_1^2 + 3h_t^2 + \lambda^2 \\ \tilde{\gamma}_Q^Q &= h_t^2 + h_b^2 - \left( \frac{1}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2 \right) \\ \tilde{\gamma}_U^U &= 2h_t^2 - \left( \frac{8}{15}g_1^2 + \frac{8}{3}g_3^2 \right) \\ \tilde{\gamma}_D^D &= 2h_b^2 - \left( \frac{2}{15}g_1^2 + \frac{8}{3}g_3^2 \right) . \end{aligned} \quad (8)$$

Note that for  $X$ -supermultiplets in the 4D boundary,  $\varepsilon_X = 1$ , the contribution of KK-modes to the anomalous dimension is the same as that of the zero modes, what explains

<sup>8</sup>We are assuming a common supersymmetric mass for all Standard Model superpartners  $M_{\text{SUSY}}$ .

<sup>9</sup>We use the renormalization scheme of Ref. [10], where the factor  $e^t$  in (7) is accompanied by  $f_\delta = X_\delta/N$ , with  $\delta = 1$ , where  $X_1 = 2$  and the factor  $N = 2$  is coming from the  $\mathbb{Z}_2$  orbifold action. We will prove that the physical Higgs mass bound is indeed independent of the chosen renormalization scheme.

the power-law behaviour in (7), while for  $X$ -fields in the bulk, with KK-modes arranged into  $N = 2$  multiplets, there is no anomalous dimension to leading order and the only contribution turns out to be the same as for  $\mu \leq M_c$ . The  $\beta$ -functions for gauge couplings in the present model are [13]

$$\begin{aligned} 16\pi^2 \beta_{g_1} &= \left( \frac{48}{5} e^t - 3 \right) g_1^3 \\ 16\pi^2 \beta_{g_2} &= (4 e^t - 3) g_2^3 \\ 16\pi^2 \beta_{g_3} &= -3 g_1^3. \end{aligned} \quad (9)$$

A glance at Eqs. (6)-(8) shows the main difference with respect to the 4D case, where only zero modes are present and the running goes up to  $M_U^{4D}$ . The 4D  $\beta$ -function for  $\lambda$  behaves as  $16\pi^2 \beta_\lambda^{4D} = b_\lambda \lambda^3 + \dots$ , with  $b_\lambda = 4$ . Were  $\beta_\lambda$  to behave similarly with respect to  $\lambda$ , but with a power-law behaviour, the bounds on  $\lambda$  we had obtained would have been similar to those in the 4D case since, as happens to the gauge couplings, the shortening in the running scale would have been compensated by the faster (power-law) running of  $\lambda$ . However, since the  $S$ -hypermultiplet lives in the bulk, and has excited modes, the  $\beta$ -function for  $\lambda$  behaves as  $16\pi^2 \beta_\lambda = \tilde{b}_\lambda \lambda^3 e^t + \dots$ , with  $\tilde{b}_\lambda = 2$ , and the running is slowed down, leading to an enhanced value of  $\lambda$  at low energy. In fact in the crude approximation of neglecting all terms in  $\beta_\lambda$ , except the  $\lambda^3$  term, one obtains an approximate relation between the low energy values of  $\lambda$  and  $\lambda_{4D}$  as,  $\lambda^2 \simeq r \lambda_{4D}^2$  with

$$r = \frac{b_\lambda}{\rho \tilde{b}_\lambda}, \quad (10)$$

where we have crudely taken  $\lambda^{-1}(M_U) = \lambda_{4D}^{-1}(M_U^{4D}) = 0$  which provides an overestimate of  $M_H$  as we will see, but accurate enough for our purposes here. This leads to an approximate value of the upper bound on  $M_H$  in terms of  $M_H^{4D}$ , the upper bound in the 4D theory, and  $M_H^{MSSM}$ , the value of the Higgs mass in the MSSM, as

$$M_H^2 \simeq r (M_H^{4D})^2 - (r-1) (M_H^{MSSM})^2. \quad (11)$$

In our case we have  $r = 2$  and using the values of  $M_H^{4D}$  ( $\simeq 140$  GeV) and  $M_H^{MSSM}$  ( $\simeq 105$  GeV), for  $\tan\beta \simeq 2$ ,  $M_{\text{SUSY}} \simeq 1$  TeV and maximal mixing in the stop sector, we find  $M_H \simeq 165$  GeV, which is, as anticipated, a little overestimate ( $\sim 2\%$ ) of the numerical value we will find ( $\simeq 162$  GeV).

In the numerical calculation we must allow for boundary conditions consistent with perturbativity in the whole range  $M_c \leq \mu \leq M_U$ . Including higher loop corrections, the  $\beta$ -functions for all the parameters, e.g.  $\lambda$ , can be decomposed as  $\beta_\lambda = \beta_\lambda^{(1)} + \beta_\lambda^{(2)} + \dots$ . We will consider the theory is perturbative at a given scale  $\mu$  if  $|\beta_\lambda^{(2)}(\mu)| \lesssim |\beta_\lambda^{(1)}(\mu)|$ . Now to leading order in powers of  $\mu$  the  $\beta$ -functions for  $\lambda$  are given by  $(4\pi)^2 \beta_\lambda^{(1)}(\mu) =$

$2\lambda^3(\mu/M_c) + \dots$ , and  $(4\pi)^4\beta_\lambda^{(2)}(\mu) = -2\lambda^5(\mu/M_c)^2 + \dots$  [22, 23]. In this case, the condition  $|\beta_\lambda^{(2)}(M_U)| \simeq |\beta_\lambda^{(1)}(M_U)|$ , for the perturbation theory to be trustable, occurs for  $\lambda(M_U) \simeq 4\pi\sqrt{M_c/M_U}$ , that we are using as boundary condition to obtain the Higgs mass bound. The result of the numerical analysis is shown in Fig. 1, where we show the absolute

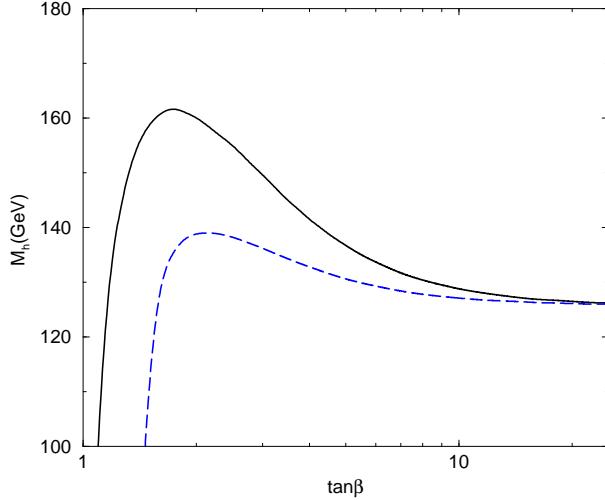


Figure 1: Upper bounds on  $M_H$  for the 5D (solid line) and 4D (dashed line) NMSSM.

upper bound on the lightest Higgs mass in the unifying model above (solid line). We have fixed, for all values of  $\tan\beta$ ,  $M_c = 4$  TeV, which is consistent with indirect bounds on  $M_c$  from electroweak precision measurements. We compare it with the corresponding curve for the 4D NMSSM unifying at  $M_U^{4D}$  and with logarithmic running of the couplings. To obtain the curves of Fig. 1 we have used initial conditions for the gauge couplings,  $h_t$  and  $h_b$ , according to the corresponding experimental values for gauge couplings and the top and bottom-quark masses. Radiative corrections are taken from the MSSM, with  $M_{\text{SUSY}} = 1$  TeV and maximal stop mixing.

If one of the Higgs fields,  $H_1$  ( $H_2$ ), is in the bulk and the other,  $H_2$  ( $H_1$ ), in the 4D boundary, one can still have the coupling (1) consistent with Eq. (5) if the singlet(s)  $\vec{S}$  is in the 4D boundary. However we will not consider this case in detail since the obtained bound is weaker than in the previous one. In fact using (7) and (8) one can see that now  $\tilde{b}_\lambda = 3$  and, assuming as before a model with  $\rho = 1$ , we get  $r = 4/3$  that yields, using (11), an absolute upper bound  $\sim 147$  GeV.

Up to this point we have considered a genuine 5D mechanism by which the lightest Higgs mass upper bound in the NMSSM unifying at a given scale  $M_U$  can be enhanced, i.e. the effect by which  $\tilde{b}_\lambda/b_\lambda < 1$ . There is also another possibility, in particular  $\rho < 1$ , that we have not yet considered. However this possibility, already existing in 4D models unifying at scales  $\ll M_U^{4D}$  [3], will not be discussed in great length. We only wish to mention how it can be achieved in the class of the considered 5D models. Assuming  $H_1$

and  $H_2$  in the 4D boundary Eq. (3) can be satisfied for models with  $\rho < 1$ . For instance, considering right-handed quarks and leptons living in the bulk,  $\Delta N_E = 2$  (two extra  $E$ -like hypermultiplets in the bulk)  $T_1 = 1$  and  $T_0 = 1/2$  (one hypermultiplet in the  $(\mathbf{1}, \mathbf{3})_0$ ), one obtains a model that unifies at one-loop as the MSSM but with  $\rho = 1/2$ . In this case  $r = 4$  and, using Eq. (11), one gets an absolute upper bound on  $M_H$  as  $\sim 208$  GeV. However, as discussed earlier, in this class of models the condition for the dominance of the KK threshold effects  $M_c \ll M_s$  can be endangered if  $\rho \ll 1$ .

A further interesting point is how robust is the present bound on the Higgs mass, shown in Fig. 1, with respect to the number of dimensions  $\delta$ <sup>10</sup> and also with respect to the regularization scheme used in Eqs. (6) through (9). For an arbitrary number of dimensions  $\delta$  and regularization scheme choice, we should replace in Eqs. (6) through (9),  $e^t \rightarrow f_\delta e^{\delta t}$ , and in (4),  $M_U/M_c \rightarrow (f_\delta/\delta)(M_U/M_c)^\delta$ , where the function  $f_\delta$  depends on the chosen regularization scheme and orbifold projection. In this way the  $\beta$ -function for  $\lambda$  behaves as  $16\pi^2\beta_\lambda = \tilde{b}_\lambda\lambda^3 f_\delta e^{\delta t} + \dots$ , the running is faster than linear if  $\delta > 1$  and also depends on the regularization scheme  $f_\delta$ . However using the resulting modified relation between  $M_U^{4D}$  and  $M_U$ ,

$$\frac{f_\delta}{\delta} \left( \frac{M_U}{M_c} \right)^\delta = \rho \log \frac{M_U^{4D}}{M_c},$$

we can see that one obtains the approximate relation at low energy,  $\lambda^2 \simeq r \lambda_{4D}^2$ , where  $r$  is given in (10), and so the relation (11), involving the lightest Higgs mass  $M_H$ , holds. In other words, a change in the number of dimensions  $\delta$  and/or the renormalization scheme  $f_\delta$ , that can modify the rate of running of all parameters of the theory, can be approximately encoded in a change of the value of the unification scale  $M_U$ , while the prediction on the upper bound of the lightest Higgs mass remains invariant.

In summary, we have considered how the presence of extra dimensions can change the absolute upper bound on the lightest Higgs mass in extensions of the MSSM with singlets. In particular we have found that if either of the Higgs or singlet multiplets live in the bulk of the extra dimension, because of the absence of wave function renormalization of  $N = 2$  hypermultiplets to leading order, the  $\beta$ -function of the Yukawa coupling involving the singlet and Higgs superfields is suppressed, leading to a slower running of the coupling and to a larger value at low scale, with the corresponding enhancement of the tree-level contribution to the Higgs mass. This effect raises the Higgs mass from  $\sim 140$  GeV in 4D models to  $\sim 165$  GeV in models with extra dimensions and some matter fields living in the bulk. The low energy behaviour of these models is like the NMSSM but with an enhanced lightest Higgs mass. This enhancement corresponds to the range of energies that will be explored in the near future and can thus have an important phenomenological impact.

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<sup>10</sup>We are assuming here that the heavy KK excitations are arranged into  $N = 2$  4D supermultiplets. Our results crucially depend on this assumption, which is naturally realized in models with  $\delta = 1, 2$ .

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